DISTORTION OF ENERGY SPECTRUM OF DECAYING ISOTROPIC TURBULENCE DUE TO SUSPENDED PARTICLES

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Analysis of previously obtained [1] dynamic equations for the correlation of the velocities of a fluid and the fine particles suspended in it indicate that in the final period of decay of isotropic turbulence the presence of suspended particles not only leads to a more rapid (exponential) damping of the fluctuations but, in the case of finite values of the fluid-particle density ratio, also results in distortion of the spectrum and reduction of the microscales of turbulence.

In the final period of decay of isotropic turbulence of a mixture of a fluid and particles whose density is higher than that of the fluid the traces of the correlation tensors are described by the system of equations [1]

$$\frac{\partial}{\partial t}V_{i,i} = 2\nu\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}V_{i,i}\right) - c\rho\,(2V_{i,i}-T_{i,i}),$$
$$\frac{\partial}{\partial t}W_{i,i} = -c\varkappa\,(2W_{i,i}-T_{i,i}) \qquad \left(c=\frac{9\nu}{2a^2}\right), \qquad (1)$$
$$\frac{\partial}{\partial t}T_{i,i} = \nu\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}T_{i,i}\right) - c\,(\rho+\varkappa)\,T_{i,i} + 2c\,(\rho W_{i,i}+\varkappa V_{i,i}).$$

Here ν is the kinematic viscosity of the fluid, a is the radius of the particles, which are assumed to be spherical, ρ is the mean volume concentration of particles in the mixture, $\varkappa$ is the fluid-particle density ratio,

$$V_{i,i} = \langle(v_i)_A\,(v_i)_B\rangle, \qquad W_{i,i} = \langle(w_i)_A\,(w_i)_B\rangle, \qquad T_{i,i} = 2\,\langle(v_i)_A\,(w_i)_B\rangle.$$

Here v_i and w_i are the components of the fluctuation velocities of fluid and particles.

System (1) was obtained on the assumption that $\rho \ll 1$, $a \ll \lambda_0$ where λ_0 is the internal scale of turbulence. Moreover, for the interaction between particles and fluid we have taken the usual Stokes expression.

We now introduce the three-dimensional energy spectrum functions

$$E\,(k,\,t) = \frac{1}{\pi}\int_0^\infty kr\sin kr V_{i,i}\,(r,\,t)\,dr,$$
$$F\,(k,\,t) = \frac{1}{\pi}\int_0^\infty kr\sin kr W_{i,i}\,(r,\,t)\,dr,$$
$$G\,(k,\,t) = \frac{1}{\pi}\int_0^\infty kr\sin kr T_{i,i}\,(r,\,t)\,dr. \qquad (2)$$

Applying the Fourier sine-transformation to the functions $rV_{i,i}(r,t)$, $rW_{i,i}(r,t)$, $rT_{i,i}(r,t)$, we reduce system (1) to the form

$$\frac{\partial E}{\partial t} = -2\nu k^2E - 2c\rho E + c\rho G,$$
$$\frac{\partial F}{\partial t} = -2c\varkappa F + c\varkappa G,$$
$$\frac{\partial G}{\partial t} = -\nu k^2G - c\,(\rho+\varkappa)\,G + 2c\rho F + 2c\varkappa E. \qquad (3)$$

System (3) describes the variation with time of the spectrum functions defined by relations (2). Note that in the absence of particles, i.e., when $\rho = 0$, the first equation of system (3) is identical with the usual dynamic energy spectrum equation, when the spectral transfer function [2] is neglected.

After transformations, the characteristic equation of system (3) takes the form

$$(\nu k^2 + c\rho + c\varkappa + \lambda)\,[(2\nu k^2 + 2c\rho + \lambda)\,(2c\varkappa + \lambda) - 4c^2\varkappa\rho] = 0.$$

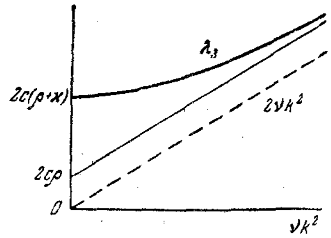


Fig. 1

The roots of this equation are

$$\lambda_1 = -\omega - 2c\varkappa, \qquad \lambda_2 = \lambda_1 + (\omega^2 + 4c^2\varkappa\rho)^{1/2},$$
$$\lambda_3 = \lambda_1 - (\omega^2 + 4c^2\varkappa\rho)^{1/2}. \qquad (4)$$

As a result, we arrive at the following fundamental system of solutions:

$$E_1 = 4c^2\varkappa\rho\; e^{\lambda_1(t-t_0)}, \quad F_1 = -4c^2\varkappa\rho\; e^{\lambda_1(t-t_0)},$$
$$G_1 = 4c\varkappa\omega\; e^{\lambda_1(t-t_0)},$$
$$E_2 = \{\omega\,[\omega - (\omega^2 + 4c^2\varkappa\rho)^{1/2}] +$$
$$+ 2c^2\varkappa\rho\}\; e^{\lambda_2(t-t_0)},$$
$$F_2 = 2c^2\varkappa^2\; e^{\lambda_2(t-t_0)}, \quad F_3 = 2c^2\varkappa^2\; e^{\lambda_3(t-t_0)},$$
$$G_2 = -2c\varkappa\,[\omega - (\omega^2 + 4c^2\varkappa\rho)^{1/2}]\; e^{\lambda_2(t-t_0)},$$
$$E_3 = \{\omega\,[\omega + (\omega^2 + 4c^2\varkappa\rho)^{1/2}] + 2c^2\varkappa\rho\}\; e^{\lambda_3(t-t_0)},$$
$$G_3 = -2c\varkappa\,[\omega + (\omega^2 + 4c^2\varkappa\rho)^{1/2}]\; e^{\lambda_3(t-t_0)}. \qquad (5)$$

In expressions (4) and (5) $\omega = \nu k^2 + c\,(\rho - \varkappa)$.

Not all the solutions of system (3) have physical significance. In fact, the quantities E and F must be essentially positive, i.e., E_i and F_i in (5) must have the same sign. Obviously, this is not fulfilled for E_1 and F_1 corresponding to the root λ_1 of the characteristic equation; hence this solution must be discarded. The functions E_2 and F_2 have the same sign only over a limited interval of variation of the wave number k. Moreover, these functions tend to zero as $\rho \to 0$ or $\varkappa \to 0$. In fact, as $\rho \to 0$ the quantity E should go over into the spectrum function for a homogeneous turbulent fluid, and as $\varkappa \to 0$ it should correspond to the solution of [1].

Here we shall confine ourselves to a consideration of the particular solution corresponding to the root λ_3 of the characteristic equation. It is easy to see that even in the case of negative ω the quantities E_3 and F_3 have the same sign. For $\omega = \min\{\omega\} = -c\,(\varkappa - \rho)$ we have

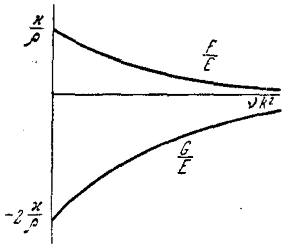
$$\min\{E_3\} \approx 2c^2\rho^2\exp\,[\lambda_3\,(t - t_0)] > 0, \qquad F_3 > 0.$$

The dependence of the damping factor and the relative values of the spectrum functions corresponding to the root λ_3 on the values of the wave number is given in Figs. 1 and 2.

Thus, we can write

$$E\,(k,\,t) = E\,(k,\,t_0)\exp\{-(\nu k^2 + c\,(\rho+\varkappa) +$$
$$+ [(\nu k^2)^2 + 2c\,(\rho-\varkappa)\,\nu k^2 + c^2\,(\rho+\varkappa)^2]^{1/2})\,(t - t_0)\}. \quad (6)$$

It is easy to see that as $\rho \to 0$ relation (6) gives the spectrum function for a homogeneous turbulent fluid [2]. As $\varkappa \to 0$ relation (6) yields the result previously obtained in [1], namely, that at $\varkappa \approx$ ≈ 0 the turbulent motion of the fluid-particle mixture is similar to the turbulent motion of the homogeneous fluid (without particles) in the sense that the presence of the particles affects only the fluctuation energy, the scales of turbulence and the structure of the energy spectrum remaining unaffected.



For large and small wave numbers we get, respectively:

$$E(k,t)\approx E(k,t_0)\exp\left\{-\left[2\nu k^2+2c\rho+\frac{2c^2\varkappa\rho}{\nu k^2}+O((\nu k^2)^{-2})\right](t-t_0)\right\}$$
$$(\nu k^2 \gg c\rho,\ \nu k^2 \gg c\varkappa), \quad (7)$$

$$E(k,t)\approx E(k,t_0)\exp\left\{-\left[\frac{2\rho\nu k^2}{\rho-\varkappa}+2c(\rho+\varkappa)+O((\nu k^2)^2)\right](t-t_0)\right\}$$
$$(\nu k^2 \ll c\rho,\ \nu k^2 \ll c\varkappa). \quad (8)$$

From (6), (7) and (8) it follows that at $\varkappa \neq 0$ the decay of isotropic turbulence of the fluid-particle mixture is accompanied by a considerable distortion of the energy spectrum as compared with the case of a homogeneous fluid. In the region of large wave numbers the presence of particles is mainly expressed in the appearance of a term describing additional exponential damping without serious distortion of the spectrum (i. e., the result of [1] relating to the case $\varkappa \approx 0$ has a universal character in this region). In the region of small wave numbers there is observed not only additional damping but a change in viscous damping, and hence distortion of the energy spectrum.

Thus, the particle effect is most important at small wave numbers. Despite the widely disseminated a priori assertions of [2], it is precisely in this region of wave numbers that the distortion of the energy spectrum is most significant, i. e., the particles favor the decay of large, rather than small eddies.

Using expression (8) and the fact that at small k the function $E(k, t_0) \sim k^4$, it is easy to obtain an expression for the trace of the correlation tensor $V_{i,i}$. We have

$$V_{i,i}(r,t)=2\int_0^\infty \frac{\sin kr}{kr}E(k,t)\,dk\sim$$
$$\sim \text{const}\, t^{-5/2}\left[3-\frac{r^2(\rho+\varkappa)}{4\nu\rho t}\right]\exp\left[-\frac{r^2(\rho+\varkappa)}{8\nu\rho t}-2c(\rho+\varkappa)t\right]. \quad (9)$$

Hence the turbulent fluctuations of the fluid in the presence of suspended particles damp according to the law

$$\langle u'^2\rangle = \text{const}\, t^{-5/2}\exp\left[-2c(\rho+\varkappa)t\right]. \quad (10)$$

At $\varkappa = 0$ expressions (9) and (10) reduce to those obtained previously for high-inertia particles [1]. Finally, we note that, as before, the correlation coefficients are described by a Gaussian curve. Thus, for the longitudinal correlation coefficient from (9) we have

$$f(r,t)=\exp\left[-\frac{r^2(\rho+\varkappa)}{8\nu\rho t}\right]. \quad (11)$$

Hence it follows that the microscales of turbulence expressed in terms of the correlation coefficients are reduced by $[(\rho + \varkappa)/\rho]^{1/2}$ times as compared with the case of a homogeneous fluid. The process of decay of turbulence of the mixture in the final period will be self-similar, as in the case of a homogeneous fluid.

We note that, if the factor $\exp[-2c(\rho + \varkappa)t]$ is neglected, Eqs. (9), (10) and (11) coincide with the corresponding expressions for the turbulence of a homogeneous fluid, if instead of the kinematic viscosity ν we introduce some effective coefficient $\nu^* = \nu\rho/(\rho + \varkappa)$.

We note further that (9) follows from the asymptotic representation of (8), valid at $\nu k^2 \ll c\rho$. Therefore in (9) and the expressions following from it passage to the limit as $\rho \to 0$ is impossible.

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